Examples of questions for the ALR fulfilling test

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(1) If it is not true that all the students of a course are tall and fat, then:

A no student is tall and no student is fat

B there are students who are not tall and there are students who are not fat

C no student is tall or no student is fat

D at least one student is not tall or at least one student is not fat

E no student is tall and fat

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(2) In a group of people there are tall tennis players and every tennis player is young. Then:

A every young person is tall

B every young person is a tennis player

C there exists a young and tall person

D there exists a young and not tall person

E every tall person is young

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(3) In a group of people every climber is young and every young person is not tall. Then:

A every person who is not tall is a climber

B there are tall people who are not climbers

C there are tall and not young people

D every tall person is not a climber

E every tall person is a climber

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(4) Given the sets A, B, and C, such that $A \subseteq C$, the set $(A \cup C) \cap (B \cup C)$ is equal to:

A $B \cup C$

B C

C B

D $A \cap B$

E $A \cap (B \cup C)$

(5) Let $A = \{0,1,2,3\}$, $B = \{1,3,5,7\}$ and $C = \{0,1,7\}$. The set $(A \cup B) \setminus (A \cap C)$ is equal to:

- A $\{5\}$
- B $\{5,7\}$
- C $\{2,3,5\}$
- D $\{2,3,5,7\}$
- E $\{0,1,2,3,5,7\}$

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(6) If A is a set with 3 elements, B is a set with 5 elements, and C is a set with 7 elements, then $(A \cup B) \cap C$

- A can be empty
- B has at least 3 elements and can have exactly 3
- C has at least 5 elements
- D has exactly 7 elements
- E can have 8 elements

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(7) Let $f(x) = \log(x^2 + 2x + 1)$. Then the natural domain of f is:

- A $\mathbf{R} \setminus \{-1\}$
- B $]0, +\infty[$
- C $[0, +\infty[$
- D $]-1, +\infty[$
- E $\mathbf{R}$

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(8) The function f defined by $f(x) = \dfrac{2}{2 - \frac{1}{x+1}}$ has as natural domain the set

- A $\mathbf{R}$
- B $\mathbf{R} \setminus \{0\}$
- C $\mathbf{R} \setminus \{-1\}$
- D $\mathbf{R} \setminus \left\{-\frac{1}{2}\right\}$
- E $\mathbf{R} \setminus \left\{-\frac{1}{2}, -1\right\}$

(9) Let

$$f : \mathbf{R} \to \mathbf{R}, \qquad f(x) = \frac{2x}{x^2+1}.$$

If $\alpha \in \mathbf{R}$, then $f(3\alpha)$ is equal to:

- A $\frac{2\alpha}{\alpha^2+1}$
- B $\frac{6\alpha}{9\alpha^2+1}$
- C $\frac{3\alpha}{9\alpha^2+1}$
- D $\frac{6\alpha}{9\alpha^2+3}$
- E $\frac{6\alpha}{3\alpha^2+1}$

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(10) Let $f, g \colon \mathbf{R} \to \mathbf{R}$ be such that, for every $x \in \mathbf{R}$, $f(x) = x + x^2$ and $g(x) = 2x^2$. Then $(f \circ g)(x)$ is equal to:

- A $2x^2 + 2x^4$
- B $2x^2 + 4x^3 + 2x^4$
- C $2x^6$
- D $2x^2 + 4x^4$
- E $2x^3 + 2x^4$

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(11) If the polynomials $P(x)$ and $Q(x)$ have degrees 8 and 3 respectively, then the division of $P(x)$ by $Q(x)$ necessarily:

- A has a quotient of degree 3 and a remainder of degree less than or equal to 2
- B has a quotient of degree 5 and a remainder of degree equal to 2
- C has a quotient of degree less than or equal to 5 and a remainder of degree less than or equal to 3
- D has a quotient of degree 5 and a remainder of degree less than 3
- E has a quotient of degree 3 and a remainder of degree less than 5

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(12) The Greatest Common Divisor of the polynomials $x^3 - x^2 - x + 1$, $1 - x^2$, and $x^3 + x^2 - x - 1$ is:

- A $x^2 - 1$
- B $x^2 + 1$
- C $x + 1$
- D $x - 1$
- E 1

(13) In the field of real numbers, the equation $3x^4 - 2x^2 - 1 = 0$

A has exactly two solutions

B has exactly three solutions

C has exactly four solutions

D has at least four solutions

E has no solutions

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(14) In the field of real numbers, the equation $\sqrt{x-1} = -(x-3)$

A has exactly two solutions

B has exactly three solutions

C has no solutions

D has 0 as a solution

E has a single solution

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(15) How many distinct real solutions does the equation $(x^3+1)(x^2+1)(x+1)^2 = 0$ have?

A none

B one

C two

D three

E four

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(16) Decreasing the triple of a real number by 5 gives the double of that number increased by 7. Then the number is equal to:

A 2

B $\frac{2}{5}$

C 12

D $\frac{12}{5}$

E 5

(17) The set of solutions of the equation $1+|x|=|1+x|$ is:

A $[0,+\infty[$

B $[0,1]$

C $[1,+\infty[$

D $]-\infty,-1]$

E $\mathbf{R}\setminus\{0\}$

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(18) The set of solutions of the inequality $\dfrac{3x-2}{x+4}>1$ is:

A $]-\infty,-4[$

B $]3,+\infty[$

C $]1,+\infty[$

D $]-\infty,-1[$

E $]-\infty,-4[\,\cup\,]3,+\infty[$

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(19) The set of solutions of the inequality $x^3+9x^2\leq 0$ is:

A $]-\infty,-9]\cup\{0\}$

B $]-\infty,-9]$

C $[-9,+\infty[$

D $]-\infty,0]$

E $[0,9]$

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(20) The set of solutions of the inequality $(x+1)(x^2+2)(x^3-3)<0$ is:

A $]1,+\infty[$

B $]-\infty,-1[\,\cup\,\left]\sqrt[3]{3},+\infty\right[$

C $\emptyset$

D $\left]\sqrt[3]{3},+\infty\right[$

E $\left]-1,\sqrt[3]{3}\right[$

(21) The set of solutions of the inequality $\dfrac{4x^2+6x-11}{x+2} \le 3x+2$ is:

A $]-\infty,-3]\cup]-2,5]$

B $]-\infty,-3]\cup[-2,5]$

C $[-3,-2[\cup]-2,5]$

D $\left]-2,-\dfrac{\sqrt{53}+3}{8}\right]\cup\left[-\dfrac{2}{3},\dfrac{\sqrt{53}-3}{8}\right]$

E $]-\infty,-2[\cup]-2,0]$

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(22) The set of solutions of the inequality $\sqrt{x^2-x} > -1$ is:

A $]-\infty,0[\cup]1,+\infty[$

B $]0,1[$

C $]0,+\infty[$

D $]-\infty,0]\cup[1,+\infty[$

E $\mathbf{R}$

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(23) Which of the following statements holds for any pair of real numbers a, b such that $ab>0$?

A $a>b>0$

B $a>-b$

C $\dfrac{a}{b}>0$

D $ab^2>0$

E $2^{ab}>2$

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(24) Consider the two lines with equations $2x+y-2=0$ and $3x-y-3=0$; they are

A parallel

B intersecting at the point $(0,1)$

C intersecting at the point $(1,0)$

D coincident

E intersecting at the point $(0,2)$

(25) For which value of the real parameter a is the line with equation $(a+3)x+y-2=0$ parallel to the line with equation $y=2x-7$?

A $a=-4$

B $a=-1$

C $a=-10$

D $a=-5$

E $a=0$

(26) The equation of the line in the plane parallel to the line with equation $x=y$ and passing through the point $(-1,-4)$ is:

A $4x-y=0$

B $x-y-3=0$

C $x-y+3=0$

D $4x-y+3=0$

E $x+y+5=0$

(27) Consider the circles with equations $x^2+y^2=1$ and $(x-1)^2+(y-1)^2-4=0$; then:

A they intersect at two points

B they are concentric

C they have the same radius

D they have no points in common

E one of the two does not intersect the x-axis

(28) The line perpendicular to the line with equation $2x+y=2$ and passing through the point $(1,0)$ has equation:

A $2y-x+1=0$

B $y-x+1=0$

C $y-2x+2=0$

D $2y+x-1=0$

E $2y-2x+2=0$

(29) The diameter AB of a circle lies on the line with equation $3x - 7y - 26 = 0$. Then the equation of the line tangent to the circle at the point $A = (-3, -5)$ is:

A $x = -3$

B $y = -5$

C $7x - 3y + 6 = 0$

D $y = \frac{3}{7}x - \frac{26}{7}$

E $7x + 3y + 36 = 0$

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(30) The lines with equations $\sqrt{\pi}\, x - \pi\sqrt{\pi}\, y + \sqrt{\pi} = 0$ and $\pi^2 y - \pi x = \pi$

A are parallel and distinct

B have exactly one point in common and are not orthogonal

C are orthogonal

D are the same line

E are not parallel and do not intersect

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(31) If $f(x) = \log_e\big(g(x)\big)$, $\forall x \in \mathbf{R}$, then:

A $f(x) > 0$, $\forall x \in \mathbf{R}$

B $\log_e\big(f(x)\big) = g(x)$, $\forall x \in \mathbf{R}$

C $g(x) \geq 1$, $\forall x \in [1, +\infty[$

D $g(x) = e^{f(x)}$, $\forall x \in \mathbf{R}$

E $\log_e\left(\frac{g(x)}{f(x)}\right) = 1$, $\forall x \in \mathbf{R}$

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(32) If x and y are real numbers, what is the product of 2^{x^2} and 2^{y^2} ?

A $2^{x^2 y^2}$

B $4^{(xy)^2}$

C $2^{x^2+y^2}$

D 2^{2xy}

E $4^{x^2+y^2}$

(33) Let $f\colon \mathbf{R} \to \mathbf{R}$, $f(x) = 5^x$. Then $f(\alpha+1) - f(\alpha)$ is equal to:

A $4 \cdot 5^{\alpha}$

B 5^{α}

C $5 \cdot 5^{\alpha}$

D 5

E 1

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(34) Which of the following statements is true for every $a, b \in \mathbf{R}$?

A $\sqrt{a^2b^2} = \sqrt{ab}\,\sqrt{ab}$

B $\sqrt{a^2+b^2} = \sqrt{ab} + \sqrt{ab}$

C $\sqrt{a^2b^2} = \sqrt{a^2}\,\sqrt{b^2}$

D $\sqrt{a^2+b^2} = \sqrt{a^2} + \sqrt{b^2}$

E $\sqrt{a^2b^2} = ab$

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(35) Which of the following statements is true?

A For every $a, b \in \mathbf{R}$ such that $a \geq 0$ and $b \geq 0$, we have $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$

B For every $a \in \mathbf{R}$, we have $\sqrt{a^2} = a$

C For every $a, b \in \mathbf{R}$ such that $a \geq 0$ and $b > 0$, we have $\sqrt{a+b} = \dfrac{\sqrt{a}}{\sqrt{b}}$

D For every $a \in \mathbf{R}$ such that $a \geq 0$, we have $\sqrt{a} > 0$

E For every $a \in \mathbf{R}$ such that $a \geq 0$, we have $\left(\sqrt{a}\right)^2 = a$

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(36) If x is a positive real number, then $\dfrac{\sqrt[3]{2x^6}}{\sqrt{2x^4}}$ is equal to:

A $\dfrac{1}{\sqrt[6]{2}}$

B 1

C $\dfrac{\sqrt[3]{2}\,x}{\sqrt{2}}$

D $\sqrt[6]{x^2}$

E $\dfrac{\sqrt[3]{2x^2}}{\sqrt{2}}$

(37) If $a = \sin(1)$ then:

A $a = \frac{\pi}{2}$

B $0 < a < 1$

C $a < 0$

D a can take infinitely many values

E $\cos^2(a) + \sin^2(1) = 1$

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(38) The diagonals of a rectangle R divide each angle of R into two angles, one being five times the other. The ratio between the longer side and the shorter side of R is equal to:

A $\tan \frac{5\pi}{12}$

B $\frac{6}{5}$

C 6

D $\ell \sin \frac{5\pi}{12}$, where ℓ is the length of each diagonal

E $\ell \tan \frac{5\pi}{12}$, where ℓ is the length of each diagonal

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(39) Knowing that $\sin \alpha = \frac{4}{5}$ and $\frac{\pi}{2} < \alpha < \pi$, which of the following statements is true?

A $\cos \alpha = \frac{3}{5}$

B $\tan \alpha = \frac{4}{3}$

C $\tan \alpha = \frac{3}{4}$

D $\cos \alpha = -\frac{2}{5}$

E $\cos \alpha = -\frac{3}{5}$

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(40) Let $\alpha \in \mathbf{R}$ such that $\cos \alpha = \frac{\sqrt{7}}{5}$ and $\sin \alpha = \frac{3\sqrt{2}}{5}$. If $\beta = \pi - \alpha$, then:

A $\cos \beta = -\frac{\sqrt{7}}{5}$ and $\sin \beta = \frac{3\sqrt{2}}{5}$

B $\cos \beta = \frac{\sqrt{7}}{5}$ and $\sin \beta = \frac{3\sqrt{2}}{5}$

C $\cos \beta = -\frac{\sqrt{7}}{5}$ and $\sin \beta = -\frac{3\sqrt{2}}{5}$

D $\cos \beta = \frac{\sqrt{7}}{5}$ and $\sin \beta = -\frac{3\sqrt{2}}{5}$

E $\cos \beta = \frac{3\sqrt{2}}{5}$ and $\sin \beta = \frac{\sqrt{7}}{5}$